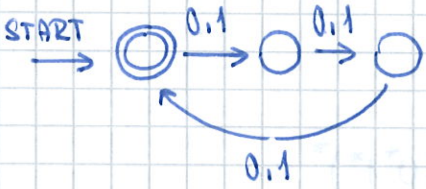
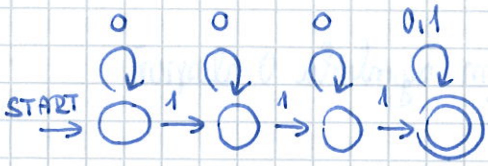
Příklad 5.1

1. $L_1 = \{ w : w \in \{0,1\}^*, 3 \mid |w| \}$ ~ délka všech slov je dělitelná 3



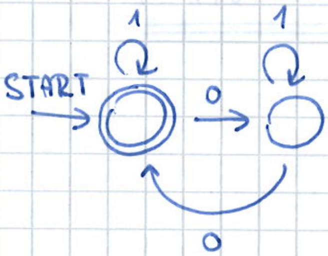
$$V_1 = ((0+1).(0+1).(0+1))^*$$

2. $L_2 = \{ w : w \in \{0,1\}^*, w$ obsahuje alespoň 3 symboly 1 $\}$



$$V_2 = 0^* 1 0^* 1 0^* 1 (0+1)^*$$

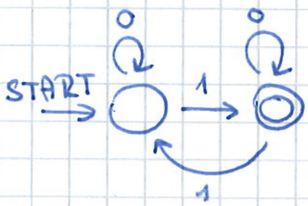
3. $L_3 = \{ w : w \in \{0,1\}^*, w$ má sudý počet symbolů 0 $\}$



$$V_3 = (1^* + 01^*0)^*$$

4. $L_4 = \{ w : w \in \{0,1\}^*, w$ má lichý počet symbolů 1 $\}$

$$V_4 = 0^* 1 \cdot (0^* + 10^*1)^*$$



5. $L_5 = \{ w : w \in \{0,1\}^*, w$ má sudý počet symbolů 0 nebo lichý počet symbolů 1 $\}$

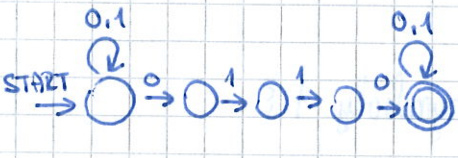
$$V_5 = (1^* + 01^*0)^* + 0^*1(0^* + 10^*1)^*$$

6. $L_6 = \{ w : w \in \{0,1\}^*,\ w$ obsahuje střídající se symboly 0 a 1 $\}$

01
010
0101

$$V_6 = 01\,(01)^*\,(0+\varepsilon)$$

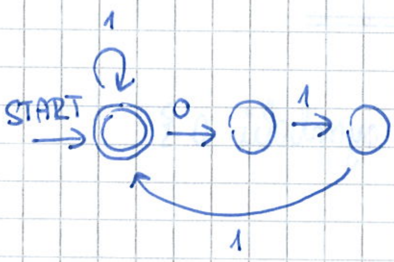
7. $L_7 = \{ w : w \in \{0,1\}^*,\ w$ obsahuje 0110 $\}$



$(0^*1^*)^*$

$$V_7 = (0+1)^*\,0110\,(0+1)^*$$

8. $L_8 = \{ w : w \in \{0,1\}^*,\ w$ má po každém symbolu 0 alespoň 2 symboly 1 $\}$



$$V_8 = (1^*(011)^*)^* = (1 + 011)^*$$

Příklad 5.2

1. $V_1 = 0^*(0^* + 1^*)$

$\quad = 0^*0^* + 0^*1^*$ A_8

$\quad = 0^* + 0^*1^*$ $0^*0^* = 0^*$

$\quad = 0^* \cdot (\varepsilon + 1)^*$ A_8

$\quad = 0^*1^*$ A_{11}

(2.) $V_2 = 11^* + 0^*0 + \varepsilon$

$= \underline{\underline{1^* + 0^*}}$

$$\left.\begin{array}{l} \varepsilon \\ 0 \\ 0\ldots0 \\ 1 \\ 1\ldots1 \end{array}\right\} \begin{array}{l} \varepsilon \\ 0\ldots0 \\ 1\ldots1 \end{array}$$

(3.) $V_3 = 0^*(1+\varepsilon)0^*(0+1)^*$

$= (0^*1 + 0^*).0^*\ (0+1)^*$

$= (0^*10^* + 0^*0^*).(0+1)^*$

$= (0^*10^* + 0^*)\cdot(0+1)^*$

$= (0^*1+\varepsilon).0^*\cdot(0+1)^*$

$= \underline{\underline{(0+1)^*}}$

$0\cdots0$ ε $0\ldots0$ cokoli
≥ 0 1 ≥ 0 cokoli

ε
1 cokoli

$0\ldots\ldots0$
$00\ldots10\ldots$ cokoli

– když odstraním neexistuje řetěz, který by neměla přijmout a přijmu a zároveň neex. řetěz, který bych měla přijmout a nepřijmu

(4.) $V_4 = (bb+aa)^*(\varepsilon + b.(b + (bb)^*baa)) + aa + bb$

$= (bb+aa)^*\cdot(\varepsilon + bb + b(bb)^*baa) + aa + bb$

$= (bb+aa)^*\cdot(\varepsilon + bb + (bb)^*bbaa) + aa + bb$

umím vygenerovat i v první části

$= (bb+aa)^*\cdot(\varepsilon + bb + (bb)^*bbaa)$

$= (bb+aa)^*\cdot(\varepsilon + (bb)^*bbaa)$

$= (bb+aa)^*\cdot(\varepsilon + bbaa)$

$= \underline{\underline{(bb+aa)^*}}$

Příklad 5.4

1.

$$x = 01^* y + 0x + 0$$
$$y = 1x + 1$$

dosadíme

$$x = 01^*(1x+1) + 0x + 0$$
$$x = 01^*1x + 01^*1 + 0x + 0$$
$$x = 01^*1x + 0x + 01^*1 + 0$$
$$x = (01^*1+0).x + 01^*1+0$$

$$x = (01^*1+0)^*(01^*1+0)$$

$$y = 1.(01^*1+0)^*(01^*1+0) + 1$$
$$y = 1.((01^*1+0)^*(01^*1+0) + \varepsilon)$$
$$y = 1.(01^*1+0)^*$$

2.

$$x = x0 + y1 + 2^* \longrightarrow x = (y1+2^*).0^*$$

$$y = x01 + y1 + 0$$

$$y = (y1+2^*).0^*01 + y1 + 0$$

$$y = y10^*01 + 2^*0^*01 + y1 + 0$$

$$y = y10^*01 + y1 + 2^*0^*01 + 0$$

$$y = y.(10^*01+1) + 2^*0^*01+0$$

$$y = (2^*0^*01+0).(10^*01+1)^*$$

$$x = ((2^*0^*01+0).(10^*01+1)^* 1 + 2^*).0^*$$

3.

$$x = 01x + 1^*y + 01 \longrightarrow x = (01)^*(1^*y+01)$$

$$y = 101y + 1x + 0$$

$$y = 101y + 1.(01)^*(1^*y+01) + 0$$

$$y = 101y + 1(01)^*1^*y + 1.(01)^*01 + 0$$

$$y = (101 + 1.(01)^*1^*)y + 1.(01)^*01+0$$

$$y = (101+1.(01)^*1^*)^*(1.(01)^*01+0)$$

$$x = (01)^*(1^*(101+1.(01)^*1^*)^*(1.(01)^*01+0) + 01)$$

(4.)

$$x = (01^* + 1)x + y$$
$$y = 11 + 1x + 00z$$
$$z = \varepsilon + x + y$$

dosazení

$$x = (01^* + 1)x + y$$
$$y = 11 + 1x + 00(\varepsilon + x + y)$$

$\longrightarrow$ $x = (01^* + 1)^* y$

$x = (0 + 1)^* y$

0
1
011..
00..

$$y = 11 + 1\cdot(0+1)^* y + 00\cdot(\varepsilon + (0+1)^* y + y)$$
$$y = 11 + 1\cdot(0+1)^* y + 00 + 00(0+1)^* y + 00y$$
$$y = 1\cdot(0+1)^* y + 00\cdot(0+1)^* y + 00y + 11 + 00$$
$$y = (1\cdot(0+1)^* + 00\cdot(0+1)^* + 00)y + 11 + 00$$
$$y = (1\cdot(0+1)^* + 00((0+1)^* + \varepsilon))y + 11 + 00$$
$$y = (1\cdot(0+1)^* + 00\cdot(0+1)^*)y + 11 + 00$$
$$y = ((1+00)\cdot(0+1)^*)y + 11 + 00$$
$$y = ((1+00)\cdot(0+1)^*)^* \cdot (11+00)$$

$$x = (0+1)^* \underbrace{((1+00)\cdot(0+1)^*)^*}_{((0+1)^*)^*}(11+00)$$

$$x = (0+1)^*(11+00)$$

$$z = \varepsilon + (0+1)^*(11+00) + ((1+00)\cdot(0+1)^*)^*(11+00)$$
$$z = \varepsilon + ((0+1)^* + \underbrace{((1+00)\cdot(0+1)^*)^*}_{((0+1)^*)^*})\cdot(11+00)$$
$$z = \varepsilon + ((0+1)^* + (0+1)^*)\cdot(11+00)$$
$$z = \varepsilon + (0+1)^* \cdot (11+00)$$

Příklad 5.5

1. $$\frac{d(10^*1)}{d1} = \frac{d1}{d1}0^*1 = \varepsilon 0^*1 = \underline{\underline{0^*1}}$$

2. $$\frac{d(01+10)^*}{d0} = \frac{d(01+10)}{d0}\cdot(01+10)^* = \left(\frac{d01}{d0}+\frac{d10}{d0}\right)(01+10)^*$$

$$= \left(\frac{d0}{d0}\cdot 1 + \frac{d1}{d0}\cdot 0\right)\cdot(01+10)^*$$

$$= (\varepsilon\cdot 1 + \emptyset . 0)\cdot(01+10)^* = \underline{\underline{1.(01+10)^*}}$$

3. $$\frac{d(010+101+0^*1+1^*0)}{d0} = \frac{d010}{d0} + \frac{d101}{d0} + \frac{d0^*1}{d0} + \frac{d1^*0}{d0} =$$

$$= 10 + \emptyset + \frac{d0^*}{d0}\cdot 1 + \frac{d1}{d0} + \frac{d1^*}{d0}\cdot 0 + \frac{d0}{d0} =$$

$$= 10 + \frac{d0}{d0}0^*\cdot 1 + \emptyset + \frac{d1}{d0}\cdot 1^*\cdot 0 + \varepsilon =$$

$$= 10 + \varepsilon . 0^* . 1 + \emptyset . 1^* . 0 + \varepsilon =$$

$$= \underline{\underline{10 + 0^*1 + \varepsilon}}$$

4. $$\frac{d(10^*1^*0)}{d100} = \frac{d}{d0}\left(\frac{d}{d0}\cdot\left(\frac{d}{d1}(10^*1^*0)\right)\right)$$

$$= \frac{d}{d0}\left(\frac{d}{d0}\cdot\left(\frac{d1}{d1}0^*1^*0\right)\right)$$

$$= \frac{d}{d0}\left(\frac{d}{d0}\cdot(0^*1^*0)\right)$$

$$= \frac{d}{d0}\cdot\left(\frac{d0^*}{d0}1^*0 + \frac{d1^*0}{d0}\right)$$

$$= \frac{d}{d0}\cdot\left(\frac{d0}{d0}0^*1^*0 + \frac{d1^*}{d0}0 + \frac{d0}{d0}\right)$$

$$= \frac{d}{d0}\cdot(0^*1^*0 + \emptyset + \varepsilon)$$

$$= \frac{d0^*}{d0}1^*0 + \frac{d1^*0}{d0} + \emptyset + \emptyset$$

$$= \underline{\underline{0^*1^*0 + \varepsilon}}$$

(5.) $$\frac{d(10^*1^*0)}{d(011)} = \frac{d}{d1}\left(\frac{d}{d1}\left(\frac{d}{d0}(10^*1^*0)\right)\right) =$$

$$= \frac{d}{d1}\left(\frac{d}{d1}\left(\frac{d1}{d0}\cdot 0^*1^*0\right)\right)$$

$$= \frac{d}{d1}\left(\frac{d}{d1}\left(\emptyset . 0^*1^*0\right)\right)$$

$$= \underline{\underline{\emptyset}}$$

Příklad 5.6

(1.) $$\int (101+010)\, d1 = 1.(101+010) = \underline{\underline{1101+1010}} + c$$

(2.) $$\int 0(1^*+0^*)\, d1 = 10(1^*+0^*) = \underline{\underline{101^*+100^*}} + c$$

(3.) $$\int 0^*(1+0^*10)\, d0 = 00^*(1+0^*10) = \underline{\underline{00^*1+00^*10}} + c$$

(4.) $$\int (101+010)\, d101 = \underbrace{\int \Big(\underbrace{\int \big(\underbrace{\int (101+010)\, d1}_{(1101+1010)}\big)\, d0}_{01101+01010}\Big)\, d1}_{\underline{\underline{101101+101010}} + c}$$

Příklad 5.7

$x = abba$

① předpony: a, ab, abb, abba

$$V = a + ab + abb + abba$$
$$V = a \cdot (\varepsilon + b + bb + bba)$$
$$V = a \cdot (\varepsilon + b \cdot (\varepsilon + b + ba))$$
$$V = a \cdot (\varepsilon + b \cdot (\varepsilon + b \cdot (\varepsilon + a)))$$

② přípony: abba, bba, ba, a

$$V = abba + bba + ba + a$$
$$V = (abb + bb + b + \varepsilon) . a$$
$$V = (ab + b + \varepsilon) . b + \varepsilon) . a$$
$$V = (((a + \varepsilon) . b + \varepsilon) . b + \varepsilon) . a$$

③ faktory

a|b|b|a|
b|b|a|
b|a|
a|

$$V = a(\varepsilon + b(\varepsilon + b(\varepsilon + a)))$$
$$+$$
$$b(\varepsilon + b(\varepsilon + a))$$
$$+$$
$$b(\varepsilon + a)$$
$$+$$
$$a$$

$\hookrightarrow$ abba

$\hookrightarrow$ bba

$\hookrightarrow$ ba

$\hookrightarrow$ a

všechny předpony všech přípon

④ podposloupnosti

$$V = (a + \varepsilon) . (b + \varepsilon)(b + \varepsilon)(a + \varepsilon)$$